Canguro Matemático Costarricense
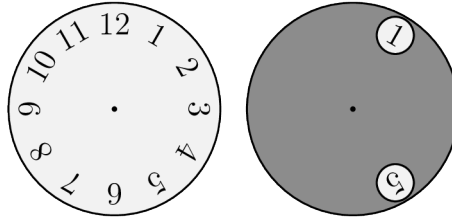

Costa Rica

Junior Test
Nineth grade

Name of the student:_____

Name of the institution:_____

Kangourou Sans Frontières
Costa Rica 2023

3 points

1. A grey circle with two holes is put on top of a clock-face, as shown.

The grey circle is turned around the centre such that the number 10 appears in one hole. Which numbers is it possible to see in the other hole?

(**A**) 2 and 6   (**B**) 3 and 7   (**C**) 3 and 6   (**D**) 1 and 9   (**E**) 2 and 7

2. Maria had to run to catch the subway, got off two stops later and then walked to school. Which of the following speed-time graphs would best represent her journey?

(**A**)   (**B**)   (**C**)   (**D**)   (**E**)
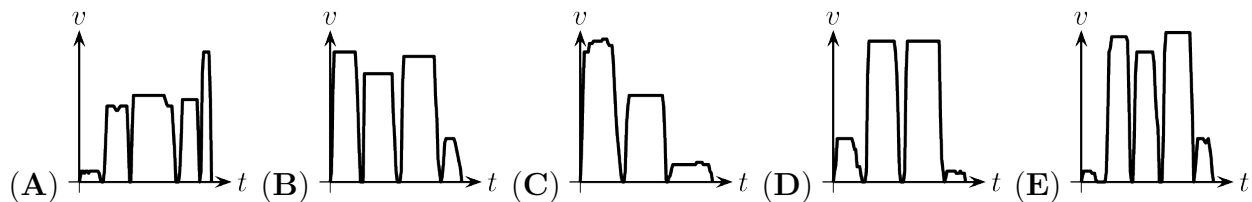
3. The positive integers m and n are both odd. Which of the following integers is also odd?

(**A**) $m(n + 1)$ (**B**) $(m + 1) \cdot (n + 1)$ (**C**) $m + n + 2$

(**D**) $m \cdot n + 2$ (**E**) $m + n$

4. A large square of side-length 10 cm contains a smaller square of side-length 4 cm, as shown in the diagram. The corresponding sides of the two squares are parallel.
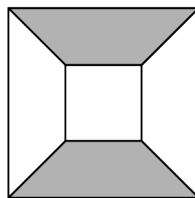
What percentage of the large square is shaded?

(**A**) 25%   (**B**) 30%   (**C**) 40%   (**D**) 42%   (**E**) 45%

5. Today is Thursday. What day will it be in 2023 days' time?

(**A**) Tuesday   (**B**) Wednesday   (**C**) Thursday   (**D**) Friday   (**E**) Saturday

6. The large rectangle in the diagram is divided into 30 equal squares, as shown. The perimeter of the shaded region is 240 cm.

What is the area of the rectangle?

(**A**) 480 cm^2 (**B**) 750 cm^2 (**C**) 1080 cm^2

(**D**) 1920 cm^2 (**E**) 2430 cm^2

7. The ages of a family of five add to 80. The two youngest are 6 and 8. What was the sum of the ages of the family seven years ago?

(**A**) 35   (**B**) 36   (**C**) 45   (**D**) 46   (**E**) 66

8. A wooden fence consists of a series of vertical planks, each joined to the next post by four horizontal planks. The first and last plank in the fence are vertical. Which of the following could be the total number of planks in the fence?

(**A**) 95   (**B**) 96   (**C**) 97   (**D**) 98   (**E**) 99

9. The diagram shows a rectangle made from three grey squares, each of area 25 cm^2, inside a larger white rectangle. Two of the vertices of the grey rectangle touch the mid-points of the shorter sides of the white rectangle and the other two vertices of the grey rectangle touch the other two sides of the white rectangle.

What is the area, in cm^2, of the white rectangle?

(**A**) 125   (**B**) 136   (**C**) 149   (**D**) 150   (**E**) 172

10. Angel has drawn a right angle. He wants to draw some straight lines coming off the vertex of the 90 grain angle, as shown, so that for any of the values 10°, 20°, 30°, 40°, 50°, 60°, 70° and 80° you can choose a pair of lines with the angle between them equal to that value.
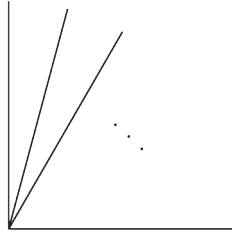
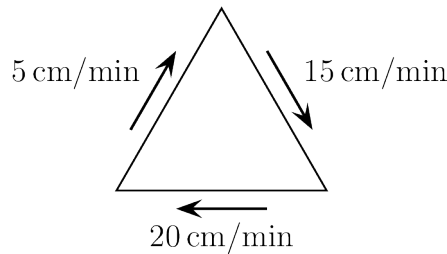What is the smallest number of lines that should be drawn?

(**A**) 2   (**B**) 3   (**C**) 4

(**D**) 5   (**E**) 6

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4 points

11. An ant is walking along the sides of an equilateral triangle. The speeds at which it travels along the three sides are 5 cm/min, 15 cm/min and 20 cm/min, as shown.

$5\,\text{cm/min}$ $15\,\text{cm/min}$

$20\,\text{cm/min}$

What is the average speed, in cm/min, at which the ant walks the whole perimeter of the triangle?

(**A**) 10   (**B**) $\dfrac{80}{11}$ (**C**) $\dfrac{180}{19}$ (**D**) 15   (**E**) $\dfrac{40}{3}$

12. Some mice live in three neighbouring houses. Last night, every mouse left its house and moved to one or the other of the other two houses, always taking the shortest route. The numbers in the diagram show the number of mice per house, yesterday and today.

yesterday   today
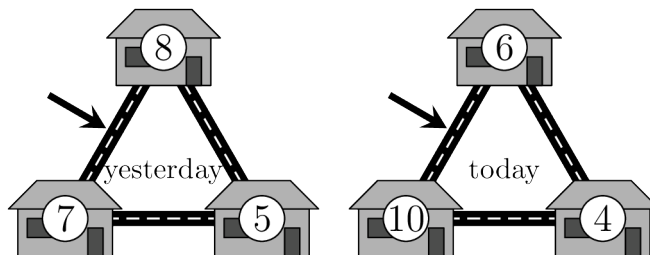
How many mice used the path shown by the arrow?

(**A**) 9   (**B**) 11   (**C**) 12   (**D**) 16   (**E**) 19

13. Jake wrote six consecutive numbers onto six white pieces of paper, one number on each piece. He stuck these bits of paper onto the top and bottom of three coins. Then he tossed these three coins three times. On the first toss, he saw the numbers 6, 7 and 8, as shown, and then coloured them red. On the second toss, the sum of the numbers he saw was 23 and on the third toss the sum was 17.
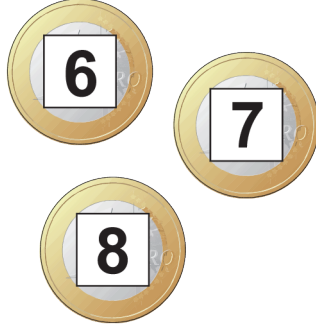
What was the sum of the numbers on the remaining three white pieces of paper?

(**A**) 18   (**B**) 19   (**C**) 23   (**D**) 24   (**E**) 30

14. The letters a and b are to be replaced by positive integers so that the equation is correct.

$$\frac{a}{5} = \frac{7}{b}$$

In how many different ways can this be done?

(**A**) 0   (**B**) 1   (**C**) 2   (**D**) 3   (**E**) 4

15. After having played 200 games of chess, my winning rate is exactly 49%. What is the smallest number of extra games I need to play to increase my winning rate to exactly 50%?

(**A**) 0   (**B**) 1   (**C**) 2   (**D**) 3   (**E**) 4

16. Jenni is trying to save water. She reduced the time she spent in her shower by a quarter. She also lowered the water pressure of her shower to reduce the rate the water comes out of the shower head by a quarter. By what fraction did Jenni reduce the total amount of water she uses for a shower?

(**A**) by $\dfrac{1}{4}$ (**B**) by $\dfrac{3}{8}$ (**C**) by $\dfrac{5}{8}$ (**D**) by $\dfrac{5}{12}$ (**E**) by $\dfrac{7}{16}$

17. The diagram shows three squares of side-length 3 cm, 5 cm and 8 cm.

What is the area, in cm², of the shaded trapezium?

(**A**) 13   (**B**) $\dfrac{55}{4}$ (**C**) $\dfrac{61}{4}$ (**D**) $\dfrac{65}{4}$ (**E**) $\dfrac{69}{4}$

18. A wire of length 95 m is cut into three pieces such that the length of each piece is 50% more than the previous piece. What is the length of the largest piece?

(**A**) 36 m   (**B**) 42 m   (**C**) 45 m   (**D**) 46 m   (**E**) 48 m

19. Points M and N are the midpoints of two sides of the rectangle.

What fraction of the area of the rectangle is shaded?

(**A**) $\dfrac{1}{6}$ (**B**) $\dfrac{1}{5}$ (**C**) $\dfrac{1}{4}$ (**D**) $\dfrac{1}{3}$ (**E**) $\dfrac{1}{2}$

20. Pentagon $ABCDE$ is divided into four triangles with equal perimeter. Triangle ABC is equilateral and AEF, DFE and CDF are three identical isosceles triangles.

What is the ratio of the perimeter of the pentagon $ABCDE$ to the perimeter of triangle ABC?

(**A**) 2/1   (**B**) 3/2   (**C**) 4/3   (**D**) 5/3   (**E**) 5/2

5 points

21. On the table there is a tower made of blocks numbered from 1 to 90. Bob takes blocks from the top of the tower, three at a time, to build a new tower, as shown.

| 90 |
| 89 |
| 88 |

| 3 |
| 2 |
| 1 |

⋮   ⋮

| 4 |
| 3 |
| 2 |
| 1 |

| 85 |
| 90 |
| 89 |
| 88 |
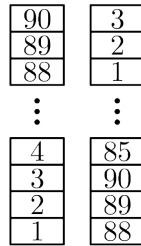
When he has finished building the new tower, how many blocks will be between the blocks numbered 39 and 40?

(**A**) 0   (**B**) 1   (**C**) 2   (**D**) 3   (**E**) 4

22. Every third step of a staircase with 2023 steps is coloured black. The first seven steps are shown in the diagram. Anita walks up the steps one at a time, starting with either her right or left foot, alternating each step.
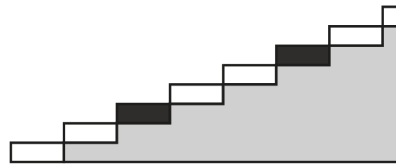
What is the smallest number of black steps she will step on with her right foot?

(**A**) 0   (**B**) 333   (**C**) 336   (**D**) 337   (**E**) 674

23. A square of side 30 cm is divided into nine identical smaller squares. The large square contains three circles with radii 5 cm (bottom right), 4 cm (top left) and 3 cm (top right), as shown.
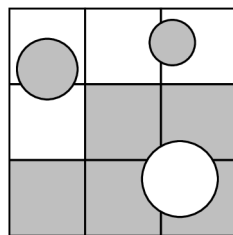
What is the area of the shaded part?

(**A**) 400 cm²   (**B**) 500 cm²   (**C**) $(400 + 50\pi)$ cm²

(**D**) $(500 - 25\pi)$ cm²   (**E**) $(500 + 25\pi)$ cm²

24. We call a two-digit number **power-less** if none of its digits can be written as an integer to a power greater than 1. For example, 53 is power-less, but 54 is **NOT** power-less since $4 = 2^2$. Which of the following is a common divisor of the smallest and the largest power-less numbers?

(**A**) 3   (**B**) 5   (**C**) 7   (**D**) 11   (**E**) 13

25. Tim calculates the mean of five different prime numbers. His answer is an integer. What is the smallest possible integer he could have obtained?

(**A**) 2   (**B**) 5   (**C**) 6   (**D**) 12   (**E**) 30

26. When it is given a list of four numbers, the Kangaroo Machine continues the list by typing the smallest non-negative integer that is different to each of the four preceding terms and then repeats this process over and over again. Jacob types in the numbers 2, 0, 2, 3, into the machine. What number will be the 2023 rd in the list?

(**A**) 0   (**B**) 1   (**C**) 2   (**D**) 3   (**E**) 4

27. When Metin's phone is fully charged, it runs out in 32 hours if he only uses it for phone-calls, in 20 hours if he only uses it for the internet, and in 80 hours if he does not use it at all. Metin gets on a train with his phone half-charged. While on the train, the time he is on the internet, the time he is making phone-calls and the time he is not using it are all the same. His phone runs out of charge just as the train reaches his destination. How many hours did the train journey take?

(**A**) 10   (**B**) 12   (**C**) 15   (**D**) 16   (**E**) 18

28. Seven different single-digit numbers are written in the circles of the diagram shown with one number in each circle. The product of the three numbers in each of the three lines of three numbers is the same.
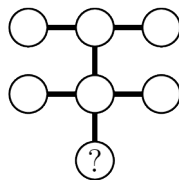
Which number is written in the circle containing the question mark?

(**A**) 2   (**B**) 3   (**C**) 4   (**D**) 6   (**E**) 8

29. Leon has drawn a closed path on a cuboid and then unfolded it to give a net. Which of the nets shown could not be the net of Leon's cuboid?
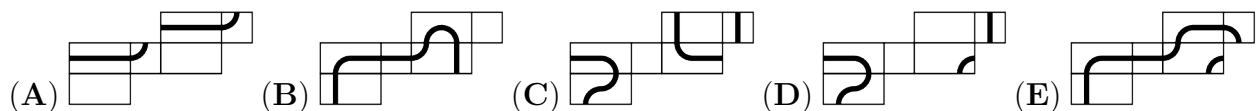
30. How many three-digit positive integers x are there, such that subtracting the sum of digits of x from x gives a three-digit number whose digits are all the same?

(**A**) 1   (**B**) 2   (**C**) 3   (**D**) 20   (**E**) 30

Name:

Institution:

01.	A	B	C	D	E

02.	A	B	C	D	E

03.	A	B	C	D	E

04.	A	B	C	D	E

05.	A	B	C	D	E

06.	A	B	C	D	E

07.	A	B	C	D	E

08.	A	B	C	D	E

09.	A	B	C	D	E

10.	A	B	C	D	E

11.	A	B	C	D	E

12.	A	B	C	D	E

13.	A	B	C	D	E

14.	A	B	C	D	E

15.	A	B	C	D	E

16.	A	B	C	D	E

17.	A	B	C	D	E

18.	A	B	C	D	E

19.	A	B	C	D	E

20.	A	B	C	D	E

21.	A	B	C	D	E

22.	A	B	C	D	E

23.	A	B	C	D	E

24.	A	B	C	D	E

25.	A	B	C	D	E

26.	A	B	C	D	E

27.	A	B	C	D	E

28.	A	B	C	D	E

29.	A	B	C	D	E

30.	A	B	C	D	E

Costa Rica