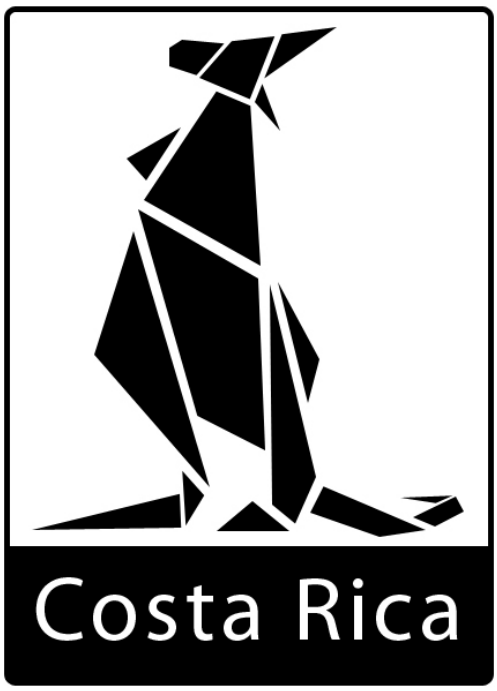
Canguro Matemático



Junior Problems

Tenth & Eleventh grade

Name: _____

Institution: _____ **Grade:** _____

Kangourou Sans Frontières

Costa Rica 2019

3 points

1. $20 \times 19 + 20 + 19 =$

(**A**) 389 (**B**) 399 (**C**) 409 (**D**) 419 (**E**) 429

2. A model train takes exactly 1 minute and 11 seconds for each round on a course. How long does it take for six rounds?

(**A**) 6 minutes 56 seconds (**B**) 7 minutes 6 seconds (**C**) 7 minutes 16 seconds
(**D**) 7 minutes 26 seconds (**E**) 7 minutes 36 seconds

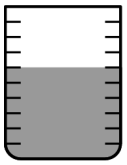
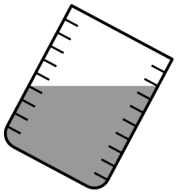
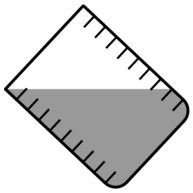
3. A barber wants to write the word SHAVE on a board in such a way that a client looking in to the mirror reads the word correctly. How should the barber write it on the board?

(**A**) SHAVE (**B**) SHAVƎ (**C**) ƎVAHS
(**D**) EVAHS (**E**) ƎVAHS

4. How many different sums of dots can you get by rolling three standard dice simultaneously?

(**A**) 14 (**B**) 15 (**C**) 16 (**D**) 17 (**E**) 18

5. Five identical glasses are filled with water. Four of them contain the same amount of water. Which one contains a different amount?

(**A**)  (**B**)  (**C**)  (**D**)  (**E**) 

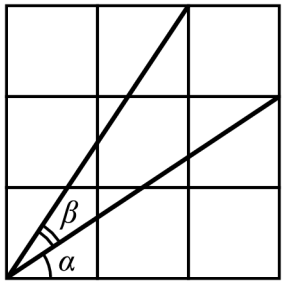
6. A park has five gates. Monica wants to enter through one gate and to exit through a different one. In how many ways can she enter and exit the park?

(**A**) 25 (**B**) 20 (**C**) 16 (**D**) 15 (**E**) 10

7. The weight of each of three kangaroos is a different whole number. The total weight of them is 97 kg. How much can the lightest of them weigh at most?

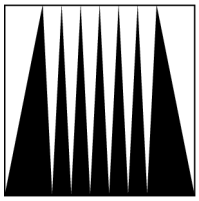
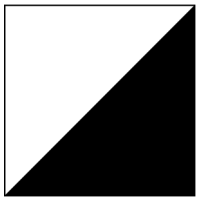
(**A**) 1 kg (**B**) 30 kg (**C**) 31 kg (**D**) 32 kg (**E**) 33 kg

8. Which of the following statements is true for the marked angles in the given figure of nine identical squares?

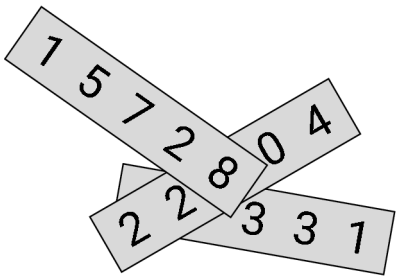


(**A**) $\alpha = \beta$ (**B**) $2\alpha + \beta = 90°$ (**C**) $\alpha + \beta = 60°$ (**D**) $2\beta + \alpha = 90°$ (**E**) $\alpha + \beta = 45°$

9. Inside each unit square a certain part has been shaded. In which square is the total shaded area the largest?

(**A**)  (**B**)  (**C**)  (**D**)  (**E**) 

10. On each of three pieces of paper a five digit number is written as shown. Three of the digits are covered. The sum of the three numbers is 57263. Which are the covered digits?



(**A**) 0, 2 and 2 (**B**) 1, 2 and 9 (**C**) 2, 4 and 9 (**D**) 2, 7 and 8 (**E**) 5, 7 and 8

4 points

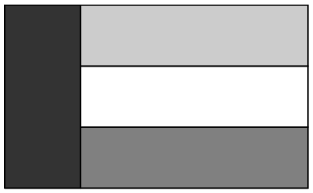
11. A square has vertices A, B, C, D labelled clockwise. An equilateral triangle is constructed with labels A, E, C labelled clockwise. What is the size of angle CBE in degrees?

(**A**) 30 (**B**) 45 (**C**) 135 (**D**) 145 (**E**) 150

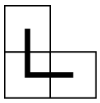
12. The numbers a, b, c, d are distinct positive integers chosen from 1 to 10. What is the least possible value $\frac{a}{b} + \frac{c}{d}$ could have?

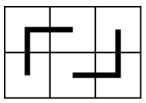
(**A**) $\frac{2}{10}$ (**B**) $\frac{3}{19}$ (**C**) $\frac{14}{45}$ (**D**) $\frac{29}{90}$ (**E**) $\frac{25}{72}$

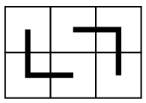
13. The flag of Kanguria is a rectangle with side lengths in the ratio 3 : 5. The flag is divided into four rectangles of equal area as shown. What is the ratio of the side lengths of the white rectangle?



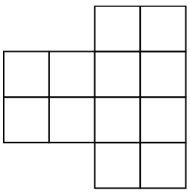
(**A**) 1 : 3 (**B**) 1 : 4 (**C**) 2 : 7 (**D**) 3 : 10 (**E**) 4 : 15

14. A 3×2 rectangle can be exactly covered by two of the L-shape figures  in two different ways as shown:





In how many different ways can the figure below be covered by the L-shape figures?



(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 48

15. The triathlon consists of swimming, running, and biking. The biking is three-quarters of the total distance; the running is one-fifth; and the swimming is 2 km. What is the total distance of this triathlon, in km?

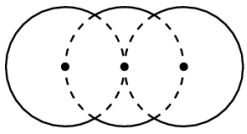
(**A**) 10 (**B**) 20 (**C**) 38 (**D**) 40 (**E**) 60

16. Some diluted juice is to be made out of concentrate and water in the ratio 1 : 7 by volume. Juice concentrate is in a 1-litre flask, and the flask is half full. What fraction of this concentrate should be used to produce 2 litres of diluted juice?

(**A**) $\frac{1}{4}$ (**B**) $\frac{1}{2}$ (**C**) $\frac{2}{7}$ (**D**) $\frac{4}{7}$

(**E**) All of the concentrate.

17. The given shape is made of parts of three equal circles of radius R that have their centres on a straight line. The middle circle passes through the centres of the other two, as shown. What is the perimeter of the shape?



(**A**) $\frac{10\pi R}{3}$ (**B**) $\frac{5\pi R}{3}$ (**C**) $\frac{2\pi R\sqrt{3}}{3}$ (**D**) $2\pi R\sqrt{3}$ (**E**) $4\pi R$

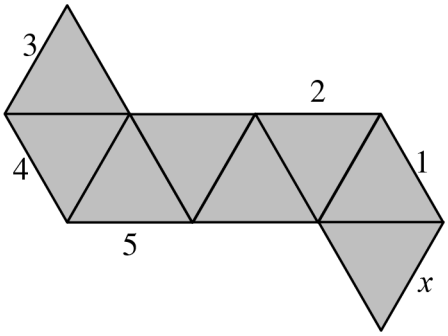
18. The seven digits of the telephone number $\overline{aaabbbb}$ add up to the two digit number $\overline{ab}$. What is the sum $a + b$?

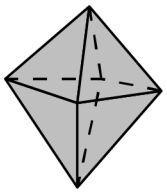
(**A**) 8 (**B**) 9 (**C**) 10 (**D**) 11 (**E**) 12

19. 60 apples and 60 pears are packed into boxes so that each box contains the same number of apples, and no two boxes contain the same number of pears. What is the largest possible number of boxes that can be packed in this way?

(**A**) 20 (**B**) 15 (**C**) 12 (**D**) 10 (**E**) 6

20. The diagram shows a net of an octahedron. When this is folded to form the octahedron, which of the labelled line segments will coincide with the line segment marked with the x?

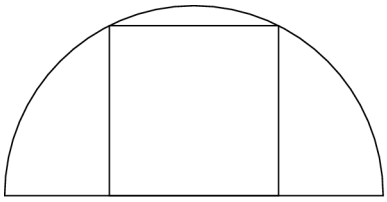




(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

5 points

21. A square has two of its vertices on a semicircle and the other two on the diameter of the semicircle as shown. The radius of the circle is 1 cm. What is the area of the square?



(**A**) $\frac{4}{5}$ cm^2 (**B**) $\frac{\pi}{4}$ cm^2 (**C**) 1 cm^2 (**D**) $\frac{4}{3}$ cm^2 (**E**) $\frac{2}{\sqrt{3}}$ cm^2

22. The sequence a_1, a_2, a_3, ... starts with $a_1 = 49$. For $n \geq 1$, the number a_{n+1} is obtained by adding 1 to the sum of the digits of a_n and then squaring the result. Thus $a_2 = (4+9+1)^2 = 196$. Determine a_{2019}.

(**A**) 121 (**B**) 25 (**C**) 64 (**D**) 400 (**E**) 49

23. The integers from 1 to 99 are written in ascending order without gaps. The sequence of digits is then divided into triplets of digits:

$$123456789101112\ldots979899 \to (123)(456)(789)(101)(112)\ldots(979)(899)$$

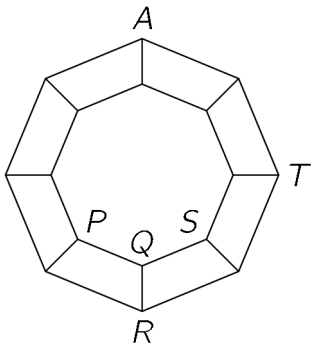
Which of the following is not one of the triplets?

(**A**) (222) (**B**) (444) (**C**) (464) (**D**) (646) (**E**) (888)

24. How many planes pass through exactly three vertices of a given cube?

(**A**) 1 (**B**) 2 (**C**) 4 (**D**) 8 (**E**) 12

25. A graph consists of 16 vertices and some edges that connect them, as in the picture. An ant is now at the vertex labelled A. At each move, it can walk from one vertex to any neighbouring vertex crawling along a connecting edge. At which of the vertices labelled P, Q, R, S, T can the ant be after 2019 moves?



(**A**) only P, R or S, not Q and T
(**B**) only P, R, S or T, not Q
(**C**) only Q
(**D**) only T
(**E**) all of these are possible

26. The positive integers a, b, and c each have three digits, and for each integer the first digit is the same as its last digit. Also $b = 2a + 1$ and $c = 2b + 1$. How many possibilities are there for the integer a?

(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 3 (**E**) more than 3

27. On each vertex of a square, one positive integer is placed. For any two numbers joined by an edge of the square, one is a multiple of the other. However, for any two diagonally opposite numbers, neither is a multiple of the other. What is the smallest possible sum of the four numbers?

(**A**) 12 (**B**) 24 (**C**) 30 (**D**) 35 (**E**) 60

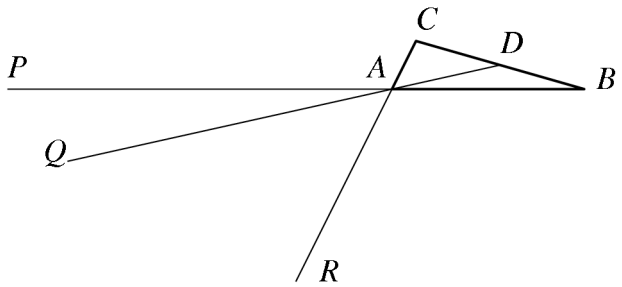
28. What is the least number of elements we have to delete from the set

$$\{10, 20, 30, 40, 50, 60, 70, 80, 90\}$$

so that the product of the elements remaining in the set is a perfect square?

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

29. Given triangle ABC of area S, let D be the midpoint of BC. Take points P, Q, R on lines AB, AD, AC, respectively, as shown in the picture, and such that $AP = 2 \cdot AB$, $AQ = 3 \cdot AD$ and $AR = 4 \cdot AC$.



What is the area of triangle PQR?

(**A**) S (**B**) $2S$ (**C**) $3S$ (**D**) $\frac{1}{2}S$

(**E**) 0 (i.e. P, Q, R are collinear).

30. If any digit of a given 4-digit number is deleted, the resulting 3-digit number is a divisor of the original number. How many 4-digit numbers have this property?

(**A**) 5 (**B**) 9 (**C**) 14 (**D**) 19 (**E**) 23

Costa Rica

Answers

Name:_____

Institution:_____ Grade: _____

01.	A	B	C	D	E
02.	A	B	C	D	E
03.	A	B	C	D	E
04.	A	B	C	D	E
05.	A	B	C	D	E
06.	A	B	C	D	E
07.	A	B	C	D	E
08.	A	B	C	D	E
09.	A	B	C	D	E
10.	A	B	C	D	E
11.	A	B	C	D	E
12.	A	B	C	D	E
13.	A	B	C	D	E
14.	A	B	C	D	E
15.	A	B	C	D	E

16.	A	B	C	D	E
17.	A	B	C	D	E
18.	A	B	C	D	E
19.	A	B	C	D	E
20.	A	B	C	D	E
21.	A	B	C	D	E
22.	A	B	C	D	E
23.	A	B	C	D	E
24.	A	B	C	D	E
25.	A	B	C	D	E
26.	A	B	C	D	E
27.	A	B	C	D	E
28.	A	B	C	D	E
29.	A	B	C	D	E
30.	A	B	C	D	E