Canguro Matemático

Student Test

Kangourou Sans Frontières

Costa Rica 2016

3 points

1. The sum of the ages of Tom and John is 23, the sum of the ages of John and Alex is 24 and the sum of the ages of Tom and Alex is 25. What is the age of the oldest one?

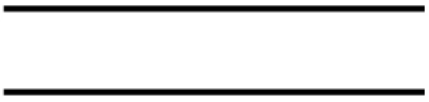
(**A**) 10 (**B**) 11 (**C**) 12 (**D**) 13 (**E**) 14

2. The sum of $\frac{1}{10} + \frac{1}{100} + \frac{1}{1000}$ is

(**A**) $\frac{3}{111}$ (**B**) $\frac{111}{1110}$ (**C**) $\frac{111}{1000}$ (**D**) $\frac{3}{1000}$ (**E**) $\frac{3}{1110}$

3. Maria wants to build a bridge across a river and knows that the shortest possible bridge from each point on one shore is always of the same length. Which of these pictures cannot be a picture of her river?

(**A**)

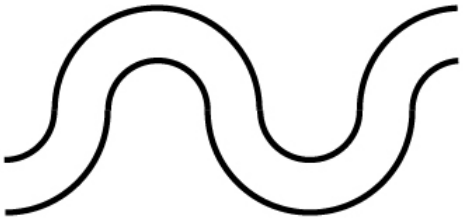


(**B**)

(**C**)

(**D**)

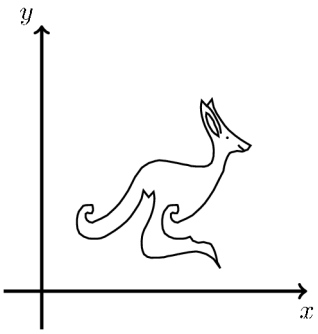
(**E**)



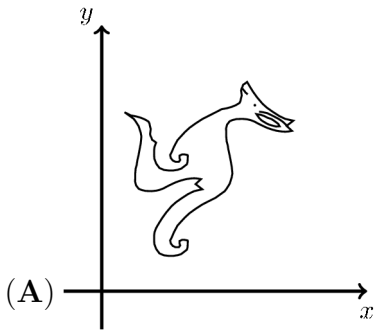
4. How many integers are greater than $2015 \cdot 2017$ but less than $2016 \cdot 2016$?

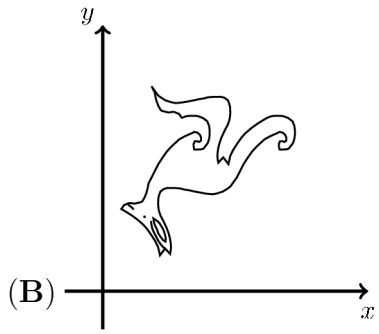
(**A**) 0 (**B**) 1 (**C**) 2015 (**D**) 2016 (**E**) 2017

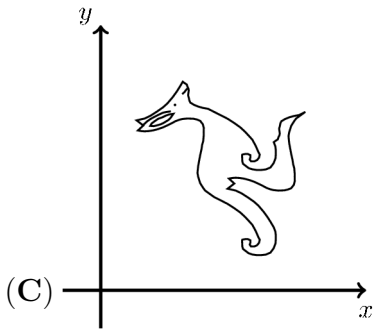
5. A set of points forms a picture of a kangaroo in the xy-plane as shown.

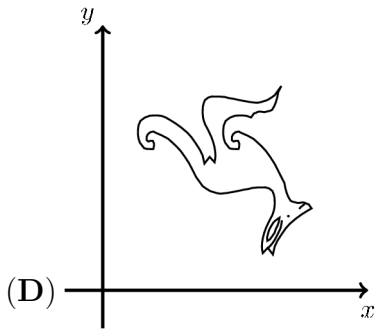


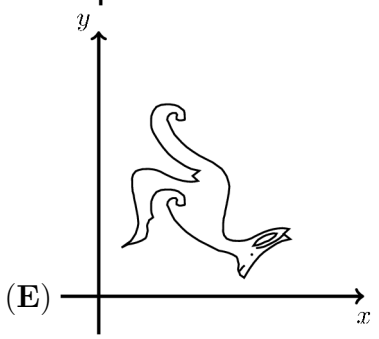
For each point the x and y coordinates are swapped. What is the result?







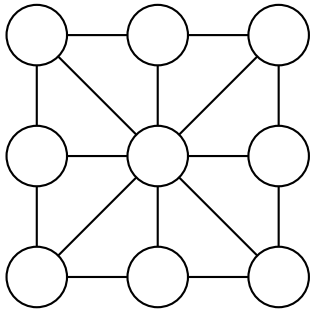




6. What is the smallest number of planes that are needed to enclose a bounded part in three-dimensional space?

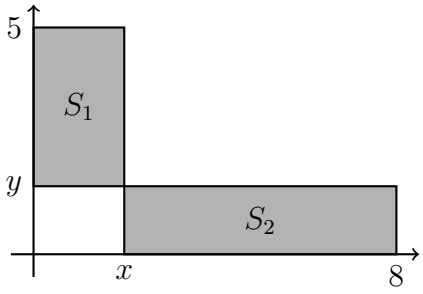
(**A**) 3 (**B**) 4 (**C**) 5 (**D**) 6 (**E**) 7

7. Diana wants to write nine integers into the circles on the diagram so that, for the eight small triangles whose vertices are joined by segments the sums of the numbers in their vertices are identical. What is the largest number of different integers she can use?



(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 5 (**E**) 8

8. The rectangles S_1 and S_2 in the picture have the same area. Determine the ratio $\frac{x}{y}$.

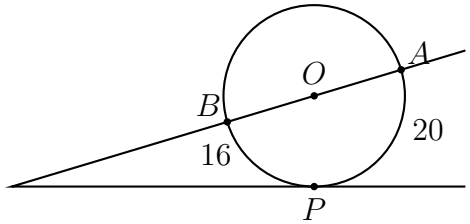


(**A**) 1 (**B**) $\frac{3}{2}$ (**C**) $\frac{4}{3}$ (**D**) $\frac{7}{4}$ (**E**) $\frac{8}{5}$

9. If $x^2 - 4x + 2 = 0$, then $x + \frac{2}{x}$ equals

(**A**) -4 (**B**) -2 (**C**) 0 (**D**) 2 (**E**) 4

10.



The lengths of arc AP and arc BP in the figure are 20 and 16, respectively. Then the value of the angle $\angle AXP$ equals

(**A**) 30° (**B**) 24° (**C**) 18° (**D**) 15° (**E**) 10°

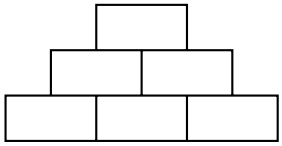
4 points

11. a, b, c, d are positive integers satisfying $a + 2 = b - 2 = c \cdot 2 = d : 2$. Which is the largest of the four numbers a, b, c and d?

(**A**) a (**B**) b (**C**) c (**D**) d

(**E**) This is not uniquely determined.

12.



In this pyramid of numbers each upper field is the product of the two fields directly underneath. Which of the following numbers cannot appear in the top field, if the three bottom fields only contain natural numbers bigger than 1?

(**A**) 56 (**B**) 84 (**C**) 90 (**D**) 105 (**E**) 220

13. What is x_4, if $x_1 = 2$ and $x_{n+1} = x_n^{x_n}$ for $n \geq 1$?

(**A**) 2^{2^3} (**B**) 2^{2^4} (**C**) $2^{2^{11}}$ (**D**) $2^{2^{16}}$ (**E**) $2^{2^{768}}$

14. In rectangle $ABCD$ the length of the side $\overline{BC}$ is half the length of the diagonal $\overline{AC}$. Let M be a point on CD such that $|\overline{AM}| = |\overline{MC}|$. What is the size of angle $\measuredangle CAM$?

(**A**) 12.5° (**B**) 15° (**C**) 27.5° (**D**) 42.5° (**E**) some other angle

15. Diana cut up a rectangle of area 2016 into 56 equal squares. The lengths of the sides of the rectangle and of the squares are integers. For how many different rectangles is it possible for her to do this?

(**A**) 2 (**B**) 4 (**C**) 6 (**D**) 8 (**E**) 0

16. On the Island of Knights and Knaves every citizen is either a Knight (who always speaks the truth) or a Knave (who always lies). During your travels on the island you meet 7 people sitting around a bonfire. They all tell you "I'm sitting between two Knaves!" How many Knaves are there?

(**A**) 3 (**B**) 4 (**C**) 5 (**D**) 6

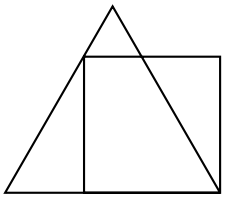
(**E**) You need more information to determine this.

17. The equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ both have real roots. It is known that the sum of squares of the roots of the first equation is equal to the sum of squares of the roots of the second one, and $a \neq b$. Then $a + b$ equals

(**A**) 0 (**B**) -2 (**C**) 4 (**D**) -4

(**E**) It is impossible to determine.

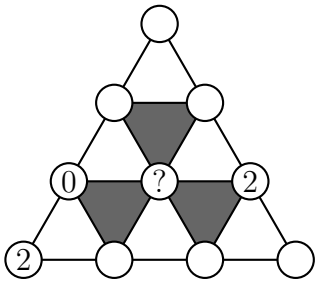
18.



If the perimeter of the square in the figure equals 4 then the perimeter of the equilateral triangle equals

(**A**) 4 (**B**) $3+\sqrt{3}$ (**C**) 3 (**D**) $3+\sqrt{2}$ (**E**) $4+\sqrt{3}$

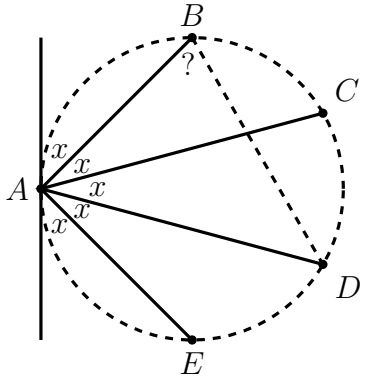
19.



Each of ten circles in the figure is marked with either 0 or 1 or 2. It is known that the sum of numbers in the vertices of any white triangle is divisible by 3, while the sum of numbers in the vertices of any gray triangle is not divisible by 3. Three of the circles are marked as shown in the figure. What numbers can be used to mark the central circle?

(**A**) Only 0 (**B**) Only 1 (**C**) Only 2 (**D**) Only 0 and 1 (**E**) Either 0 or 1 or 2

20.



Betina draws five points A, B, C, D and E on a circle as well as the tangent to the circle at A, such that all five angles marked with x are equal. (Note that the drawing is not to scale.) How large is the angle $\angle ABD$?

(**A**) 66° (**B**) 70.5° (**C**) 72° (**D**) 75° (**E**) 77.5°

5 points

21. How many different solutions are there to the equation

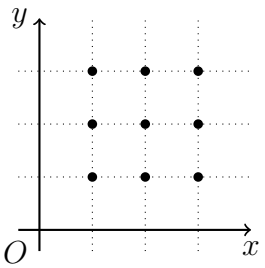
$$\left(x^2-4x+5\right)^{x^2+x-30}=1$$

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) infinitely many

22. A quadrilateral contains an inscribed circle (i.e. a circle tangent to the four sides of the quadrilateral). The ratio of the perimeter of the quadrilateral to that of the circle is 4:3. Then the ratio of the area of the quadrilateral to that of the circle is

(**A**) $4 : \pi$ (**B**) $3\sqrt{2} : \pi$ (**C**) $16 : 9$ (**D**) $\pi : 3$ (**E**) $4 : 3$

23.



How many quadratic functions in x have a graph passing through at least 3 of the marked points?

(**A**) 6 (**B**) 15 (**C**) 19 (**D**) 22 (**E**) 27

24. In a right-angled triangle ABC (right angle at A) the bisectors of the acute angles intersect at point P. If the distance from P to the hypotenuse is $\sqrt{8}$, what is the distance from P to A?

(**A**) 8 (**B**) 3 (**C**) $\sqrt{10}$ (**D**) $\sqrt{12}$ (**E**) 4

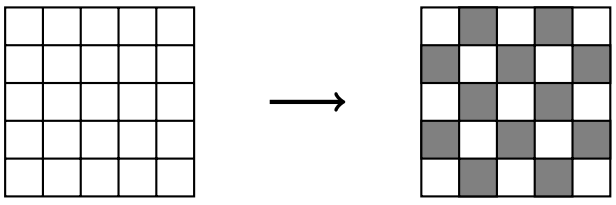
25. Three three-digits numbers are formed from digits from 1 to 9 (each digit is used exactly once). Which of the following numbers couldn't be equal to the sum of this three numbers?

(**A**) 1500 (**B**) 1503 (**C**) 1512 (**D**) 1521 (**E**) 1575

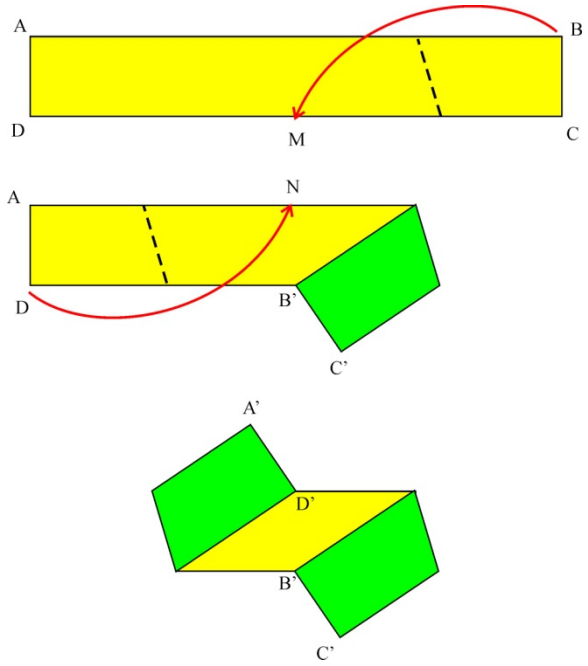
26. A cube is dissected into 6 pyramids by connecting a given point in the interior of the cube with each vertex of the cube. The volumes of five of these pyramids are 2, 5, 10, 11 and 14. What is the volume of the sixth pyramid?

(**A**) 1 (**B**) 4 (**C**) 6 (**D**) 9 (**E**) 12

27. Consider a 5×5 square divided into 25 cells. Initially all its cells are white. In each move it is allowed to change the color of any three consecutive cells in a row or in a column to the opposite color (i.e. white cells become black and black ones become white). What is the smallest possible number of moves needed to obtain the chessboard coloring shown in the figure?



(**A**) less than 10 (**B**) 10 (**C**) 12
(**D**) more than 12 (**E**) It is impossible to do.



28.

A rectangular strip $ABCD$ of paper 5 cm wide and 50 cm long is yellow on one side and green on the other. Folding the strip, Cristina makes the vertex B coincide with the midpoint M of the side CD. Folding again, she makes the vertex D coincide with the midpoint N of the side AB. What is the area in cm^2 of the visible yellow part of the strip in the picture?

(**A**) 50 (**B**) 60 (**C**) 62.5 (**D**) 100 (**E**) 125

29. Ann chose a positive integer n and wrote down the sum of all positive integers from 1 to n. A prime number p divides the sum, but not any of the summands. Which of the following could be $n + p$?

(**A**) 217 (**B**) 221 (**C**) 229 (**D**) 245 (**E**) 269

30. The positive integer N has exactly six distinct (positive) divisors including 1 and N. The product of five of these is 648. Which one of the following is the sixth divisor of N?

(**A**) 4 (**B**) 8 (**C**) 9 (**D**) 12 (**E**) 24

Costa Rica

Answers

Name:_____

Institution:_____ Grade: _____

01.	A	B	C	D	E
02.	A	B	C	D	E
03.	A	B	C	D	E
04.	A	B	C	D	E
05.	A	B	C	D	E
06.	A	B	C	D	E
07.	A	B	C	D	E
08.	A	B	C	D	E
09.	A	B	C	D	E
10.	A	B	C	D	E
11.	A	B	C	D	E
12.	A	B	C	D	E
13.	A	B	C	D	E
14.	A	B	C	D	E
15.	A	B	C	D	E

16.	A	B	C	D	E
17.	A	B	C	D	E
18.	A	B	C	D	E
19.	A	B	C	D	E
20.	A	B	C	D	E
21.	A	B	C	D	E
22.	A	B	C	D	E
23.	A	B	C	D	E
24.	A	B	C	D	E
25.	A	B	C	D	E
26.	A	B	C	D	E
27.	A	B	C	D	E
28.	A	B	C	D	E
29.	A	B	C	D	E
30.	A	B	C	D	E