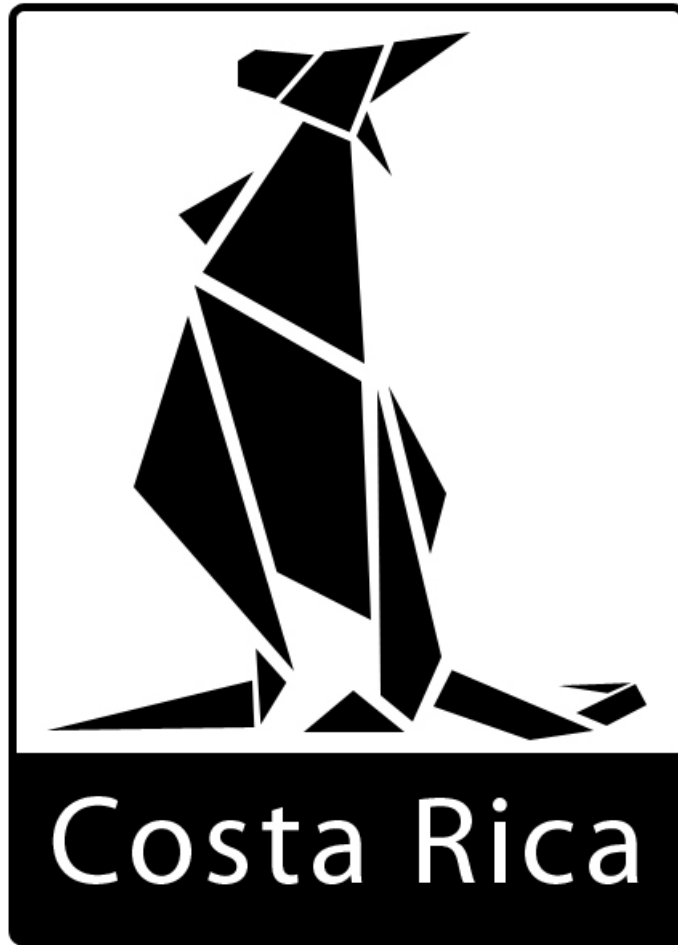


Canguro Matemático



Junior Test
Ninth grade

Kangourou Sans Frontières

Costa Rica 2016

3 points

1. The mean of four numbers is 9. What is the fourth number if three of the numbers are 5, 9 and 12?

- (A) 6 (B) 8 (C) 9 (D) 10 (E) 36

2. Which of the following numbers is the closest to the result of $\frac{17 \times 0.3 \times 20.16}{999}$?

- (A) 0.01 (B) 0.1 (C) 1 (D) 10 (E) 100

3. On a test consisting of 30 questions, Ruth had 50% more right answers than she had wrong answers. Each answer was either right or wrong. How many correct answers did Ruth have, assuming she answered all questions?

- (A) 10 (B) 12 (C) 15 (D) 18 (E) 20

4. In a coordinate system four of the following points are the vertices of a square. Which point is not a vertex of this square?

- (A) $(-1; 3)$ (B) $(0; -4)$ (C) $(-2; -1)$ (D) $(1; 1)$ (E) $(3; -2)$

5. When the positive integer x is divided by 6, the remainder is 3. What is the remainder when $3x$ is divided by 6?

- (A) 4 (B) 3 (C) 2 (D) 1 (E) 0

6. How many weeks are the same as 2016 hours?

- (A) 6 (B) 8 (C) 10 (D) 12 (E) 16

7. Little Lucas invented his own way to write down negative numbers before he learned the usual way with the negative sign in front. Counting backwards, he'd write: ... 3, 2, 1, 0, 00, 000, 0000, ... What is the result of $000 + 0000$ in his notation?

- (A) 1 (B) 00000 (C) 000000 (D) 0000000 (E) 00000000

8. I have some strange dice: the faces show the numbers 1 to 6 as usual, except that the odd numbers are negative ($-1, -3, -5$ in place of 1, 3, 5). If I throw TWO such dice, which of these totals can not be achieved?

- (A) 3 (B) 4 (C) 5 (D) 7 (E) 8

9. How many times do two directly adjacent letters have to be exchanged in order to change the word VELO step by step into the word LOVE ?

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

10. Sven wrote five different one-digit positive integers on a blackboard. He discovered that no sum of any two numbers is equal to 10. Which of the following numbers did Sven definitely write on the blackboard?

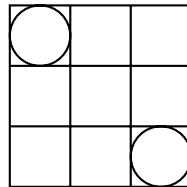
- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

4 points

11. Let $a + 5 = b^2 - 1 = c^2 + 3 = d - 4$. Which one of the numbers a, b, c, d is the largest?

- (A) a (B) b (C) c
 (D) d (E) impossible to determine

12. The 3×3 table is divided into 9 unit squares, and two circles are inscribed in two of them (see picture). What is the distance between the two circles?

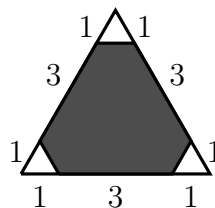


- (A) $2\sqrt{2} - 1$ (B) $\sqrt{2} + 1$ (C) $2\sqrt{2}$ (D) 2 (E) 3

13. In a tennis tournament on a knock-out basis, six of the results of the quarter-finals, the semi-finals and the final were (not necessarily in this order): Bella beat Ann, Celine beat Donna, Gina beat Holly, Gina beat Celine, Celine beat Bella and Emma beat Farah. Which result is missing?

- (A) Gina beat Bella (B) Celine beat Ann (C) Emma beat Celine
 (D) Bella beat Holly (E) Gina beat Emma

14. What percent of the area of the triangle is shaded in the figure?



- (A) 80% (B) 85% (C) 88% (D) 90%
 (E) impossible to determine

15. Eight unmarked envelopes contain the numbers: 1, 2, 4, 8, 16, 32, 64, 128. Eve chooses a few envelopes randomly. Alie takes the rest. Both sum up their numbers. Eve's sum is 31 more than Alie's. How many envelopes did Eve take?

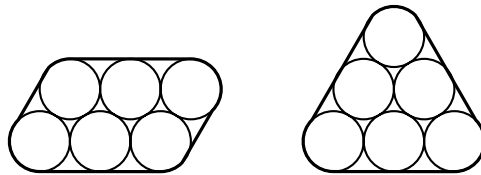
- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

16. Jill is making a magic multiplication square using the numbers 1, 2, 4, 5, 10, 20, 25, 50 and 100. The products of the numbers in each row, in each column and in the two diagonals should all be the same. In the figure you can see how she has started. Which number should Jill place in the cell with the question mark?

20	1	
		?

- (A) 2 (B) 4 (C) 5 (D) 10 (E) 25

17. Jack wants to hold six circular pipes with diameter 2 cm together by a rubber band. He decided between two options as shown in the picture. Compare the lengths of the rubber bands.



- (A) In the left picture it is π cm shorter. (B) In the left picture it is 4 cm shorter.
 (C) In the right picture it is π cm shorter. (D) In the right picture it is 4 cm shorter.
 (E) Both have the same length.

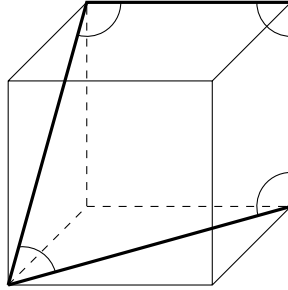
18. Peter wants to colour the cells of a 3×3 square in such a way that each of the rows, the columns and both diagonals have three cells of three different colours. What is the least number of colours Peter could use?

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

19. There are 2016 kangaroos, each of them is either grey or red and at least one of them is grey and at least one is red. For every kangaroo K we compute the fraction of the number of kangaroos of the other colour divided by the number of kangaroos of the same colour as K (including K). Find the sum of the fractions of all 2016 kangaroos.

- (A) 2016 (B) 1344 (C) 1008
 (D) 672 (E) more information is needed

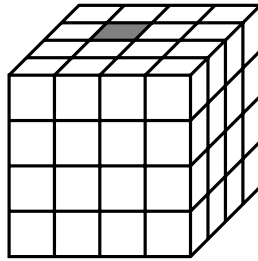
20. The picture shows a cube with four marked angles. What is the sum of these angles?



- (A) 315° (B) 330° (C) 345° (D) 360° (E) 375°

5 points

21. The cube below is divided into 64 small cubes. Exactly one of the cubes is grey. On the first day, the grey cube changes all its neighbouring cubes to grey (two cubes are neighbours if they have a common face). On the second day, all the grey cubes do the same thing. How many grey cubes are there at the end of the second day?

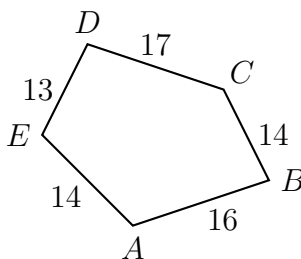


- (A) 11 (B) 13 (C) 15 (D) 16 (E) 17

22. Several different positive integers are written on a blackboard. The product of the smallest two of them is 16. The product of the largest two is 225. What is the sum of all the integers?

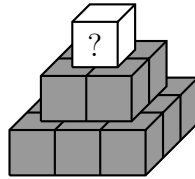
- (A) 38 (B) 42 (C) 44 (D) 58 (E) 243

23. The diagram shows a pentagon. Sonia draws five circles with centres A, B, C, D, E such that the two circles on each side of the pentagon touch. The lengths of the sides of the pentagon are given. Which point is the centre of the largest circle that she draws?



- (A) A (B) B (C) C (D) D (E) E

24. Katie writes a different positive integer on each of the fourteen cubes in the pyramid. The sum of the nine integers written on the bottom cubes is equal to 50. The integer written on each other cube is equal to the sum of the integers written on the four cubes underneath it. What is the greatest possible integer that can be written on the top cube?

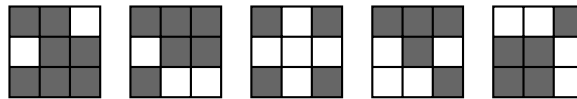


- (A) 80 (B) 98 (C) 104 (D) 110 (E) 118

25. A train has five carriages, each containing at least one passenger. Two passengers are said to be "neighbours" if either they are in the same carriage or they are in two adjacent carriages. Each passenger has either exactly five or exactly ten "neighbours". How many passengers are there in the train?

- (A) 13 (B) 15 (C) 17 (D) 20
 (E) There is more than one possibility.

26. A $3 \times 3 \times 3$ cube is built from 15 black cubes and 12 white cubes. Five faces of the larger cube are shown.



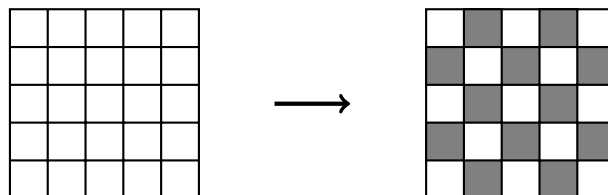
Which of the following is the sixth face of the large cube?

- (A) (B) (C) (D) (E)

27. What is the largest possible remainder that can be obtained when a two-digit number is divided by the sum of its digits?

- (A) 13 (B) 14 (C) 15 (D) 16 (E) 17

28. A 5×5 square is divided into 25 cells. Initially all its cells are white, as shown on the left. Neighbouring cells are those that share a common edge. On each move you can choose two neighbouring cells so they have their colours changed to the opposite colour (e.g. white cells become black and black ones become white). What is the minimum number of moves required in order to obtain the chess-like colouring shown on the right?



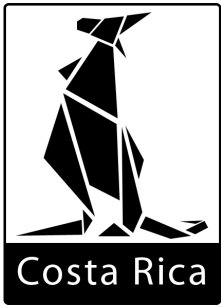
- (A) 11 (B) 12 (C) 13 (D) 14 (E) 16

29. It takes 4 hours for a motorboat to travel downstream from X to Y. To return upstream from Y to X it takes the motorboat 6 hours. How many hours would it take a wooden log to be carried from X to Y by the current, assuming it is unhindered by any obstacles?

- (A) 5 (B) 10 (C) 12 (D) 20 (E) 24

30. In the Kangaroo republic each month consists of 40 days, numbered 1 to 40. Any day whose number is divisible by 6 is a holiday, and any day whose number is a prime is a holiday. How many times in a month does a single working day occur between two holidays?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5



Answers

Name: _____

Institution: _____

Grade: _____

01. A B C D E

02. A B C D E

03. A B C D E

04. A B C D E

05. A B C D E

06. A B C D E

07. A B C D E

08. A B C D E

09. A B C D E

10. A B C D E

11. A B C D E

12. A B C D E

13. A B C D E

14. A B C D E

15. A B C D E

16. A B C D E

17. A B C D E

18. A B C D E

19. A B C D E

20. A B C D E

21. A B C D E

22. A B C D E

23. A B C D E

24. A B C D E

25. A B C D E

26. A B C D E

27. A B C D E

28. A B C D E

29. A B C D E

30. A B C D E